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$\mathcal{N}=1,2$ supersymmetric vacua of IIA supergravity and $\mathrm{SU}(2)$ structures

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Abstract: We consider backgrounds of (massive) IIA supergravity of the form of a warped product $M_{1,3} \times_\omega X_6$, where X_6 is a six-dimensional compact manifold and $M_{1,3}$ is AdS_4 or a four-dimensional Minkowski space. We analyse conditions for $\mathcal{N}=1$ and $\mathcal{N}=2$ supersymmetry on manifolds of SU(2) structure. We prove the absence of solutions in certain cases.

Keywords: Superstring Vacua, Supergravity Models.

Contents

1. Introduction and summary

String theory compactifications in the presence of fluxes possess a number of phenomenologically attractive features, a fact which has led to their intensive study in recent years. String theory is approximated in the low-energy effective-theory limit by supergravity and in many situations of physical interest it has proven extremely fruitful to study the properties of supersymmetric supergravity solutions. The presence of some amount of unbroken supersymmetry above a certain low (compared to the Planck mass) energy scale is desirable, if one wishes to avoid issues of stability.

The subject of supersymmetric supergravity compactifications with fluxes is a particularly old one. Nevertheless, it has only recently been realized (starting with [1]; see [2] for a recent review and references) that the machinery of G-structures can be a powerful tool in classifying and constructing supergravity solutions. From this point of view, a G-structure is the natural generalization of the concept of special holonomy to the case where nontrivial fluxes, i.e. nonzero vevs of the antisymmetric tensor fields, are present.

In [3] we presented a classification of $\mathcal{N} = 1$ supersymmetric solutions of IIA supergravity of the form of a warped product $AdS_4 \times_\omega X_6$, where X_6 is a six-dimensional compact manifold of SU(3) structure, generalizing the work of [4, 5]. The manifold X_6 was constrained to be 'half-flat' of a certain type. For further related work on IIA compactifications from the point of view of the four-dimensional effective field theory see [6 − 12]. Type IIA compactifications have also been considered in the context of G-structures in [13 − 17]. The recent paper [18] analyzes type II $\mathcal{N} = 1$ supersymmetry using the concept of generalized G-structures — we will come back to this in the next paragraph. Supersymmetric AdS_4 solutions are of additional interest as they are expected to be dual to certain three-dimensional superconformal field theories [19, 20].

As will be explained in the following, for the backgrounds considered here $\mathcal{N} = 1$ supersymmetry implies that the Majorana-Weyl supersymmetry parameter ϵ is of the form

$$\epsilon = \theta_+ \otimes (\alpha\eta_{1+} + \delta\eta_{2-}) + \text{c.c.}\,, \tag{1.1}$$

where α, δ, are functions on X_6, η_{1+} (η_{2-}) is a globally-defined, chiral (antichiral) unimodular spinor (and therefore *nowhere-vanishing*) on X_6 and θ is a Killing spinor of $M_{1,3}$.[1] The existence of η_1 implies that the structure group of X_6 is reduced to SU(3). If in addition $\eta_{1,2}$ are nowhere-parallel, the structure group is further reduced to SU(2). Relaxing the condition that $\eta_{1,2}$ should be linearly-independent everywhere on X_6 would lead to a situation which can be thought of as a so-called 'generalized SU(3) structure 'on X_6' [12, 16, 18, 21 − 23]: at generic points in X_6 the two SU(3) structures associated with each of the two internal spinors have a common subgroup, which defines an SU(2) structure on X_6. However, at the points where the two spinors become parallel the structure group collapses to SU(3).

Supersymmetric vacua on manifolds of SU(2) structure restrict the choice for the fluxes similar to their SU(3)-structure counterparts. In addition, the requirement of an SU(2) structure imposes a strong constraint on the internal manifold[2] and one may hope that a classification can proceed much more explicitly in this case. However the situation is much more difficult to analyze in practice, and this subject has received much less attention in the literature, because of the multitude of flux-components which arise in decomposing the supergravity fields in terms of irreducible SU(2) representations.

In the present paper we examine $\mathcal{N} = 1$ supersymmetric type IIA vacua in the case where X_6 is a compact manifold of SU(2) structure. In [3] we considered the case $\eta_1 = \eta_2$. Here we will consider the other 'extreme' case where $\eta_{1,2}$ are everywhere orthogonal.[3] In

[1] $\eta_{1,2+}$ are related to $\eta_{1,2-}$ by complex conjugation. Equation (1.1) represents one linear combination of $\eta_{1,2}$ and corresponds to $\mathcal{N} = 1$ in $d = 4$.

[2] A necessary and sufficient condition for the structure group of a manifold X_6 of SU(3) structure to further reduce to SU(2), is the existence of a globally-defined nowhere-vanishing vector field on X_6. This is equivalent to the requirement that the Euler characteristic vanish, $\chi(X_6) = 0$.

[3] This is more restrictive than requiring that $\eta_{1,2}$ in (1.1) be nowhere-parallel.

addition, we look for solutions with nonzero Romans' mass. We reformulate the supersymmetry conditions in terms of SU(2) structures in section 5. In search for explicit solutions we make some further simplifying assumptions; namely, we set all nonscalar (in the sense of irreducible SU(2) representations) fluxes to zero and we take the dilaton to be constant. This is a consistent truncation which, however, turns out to be too stringent: as we will see there do not exist any supersymmetric vacua of this type.

A related $\mathcal{N} = 1$ type IIA vacuum was constructed in [5]. In that case the manifold X_6 was taken to be conformally R^6 and therefore noncompact, allowing for non-constant harmonic functions. Taking X_6 to be T^6 instead in the solution of [5], would imply that the warp factors and the dilaton are constant and that the Romans' mass and all the fluxes are zero. The solution would therefore degenerate to $R^{1,3} \times T^6$, in agreement with our conclusion above. Type IIA supersymmetric compactifications to Minkowski space on manifolds of SU(2) structure have also been considered in [15]. However, all ten-dimensional vacua in that paper arise upon reduction of eleven-dimensional supergravity solutions and are therefore unrelated to the present work.[4]

In section 6 we proceed to examine the case of $\mathcal{N} = 2$ supersymmetric (warped) AdS_4 vacua. Rather than considering the most general spinor Ansatz, we will take the two Majorana-Weyl supersymmetry parameters $\epsilon_{1,2}$ to be of the form

$$\begin{aligned} \epsilon_1 &= \theta_+ \otimes (\alpha\eta_{1+} + \beta\eta_{1-}) + \text{c.c.} \\ \epsilon_2 &= \theta_+ \otimes (\gamma\eta_{2+} + \delta\eta_{2-}) + \text{c.c.}\,, \end{aligned} \tag{1.2}$$

where α, β, γ, δ, are functions on X_6 and $\eta_{1,2}$ are globally-defined, unimodular spinors on X_6. In addition we take $\eta_{1,2}$ to be orthogonal to each other. Requiring $M_{1,3}$ to be AdS_4 implies $\alpha = \beta$, $\gamma = \delta$. As we show in section 5, this requirement is again too stringent and in fact there do not exist any $\mathcal{N} = 2$ IIA vacua of this type.

Note that for an admissible vacuum, except for the supersymmetry conditions we also require the supergravity equations of motion and the Bianchi identities for the forms to be satisfied. More generally, the no-go theorems of this paper could be by-passed by introducing additional sources, for example orientifolds, which would modify the equations-of-motion. Alternatively one may consider singular and/or noncompact 'internal' manifolds, higher-order stringy corrections, etc. We also emphasize that, due to the technical complexity of the task, we have not been able to analyze the most general spinor Ansatz leading to SU(2) structures. We hope to report on that in the future.

The outline of the remainder of the paper is as follows: In the next section we review some useful facts about (Romans') IIA supergravity. Our Ansatz for the ten-dimensional $\mathcal{N} = 1$ background and the corresponding reduction of the supersymmetry conditions are considered in section 3. Section 4 contains a brief review of SU(3) and SU(2) structures in six dimensions. The main analysis of our Ansatz for $\mathcal{N} = 1$ supersymmetric vacua is contained in section 5. Section 6 contains the analysis of our $\mathcal{N} = 2$ Ansatz. Most of the technical details and some further useful formulæ are relegated to the four appendices.

[4]Recall that for nonzero mass parameter, as is the case here, Romans' supergravity has no Poincaré-invariant lift to eleven dimensions; see [24] for a recent discussion.

2. Massive IIA

For the sake of completeness, in this section we note some known facts about Romans' ten-dimensional supergravity. Our notation and conventions are as in [3] to which the reader is referred for further details.

The equations of motion for the bosonic fields of massive IIA supergravity are [25]

$$
\begin{aligned}
0 &= R_{MN} - \frac{1}{2}\nabla_M\phi\nabla_N\phi - \frac{1}{12}e^{\phi/2}G_{MPQR}G_N{}^{PQR} + \frac{1}{128}e^{\phi/2}g_{MN}G^2 - \\
&\quad - \frac{1}{4}e^{-\phi}H_{MPQ}H_N{}^{PQ} + \frac{1}{48}e^{-\phi}g_{MN}H^2 - \\
&\quad - 2m^2e^{3\phi/2}B'_{MP}B'_N{}^P + \frac{m^2}{8}e^{3\phi/2}g_{MN}(B')^2 - \frac{m^2}{4}e^{5\phi/2}g_{MN} \qquad (2.1)\\
0 &= \nabla^2\phi - \frac{1}{96}e^{\phi/2}G^2 + \frac{1}{12}e^{-\phi}H^2 - \frac{3m^2}{2}e^{3\phi/2}(B')^2 - 5m^2e^{5\phi/2} \qquad (2.2)\\
0 &= d(e^{-\phi} * H) - \frac{1}{2}G\wedge G + 2m\ e^{\phi/2}B'\wedge *G + 4m^2e^{3\phi/2} * B' \qquad (2.3)\\
0 &= d(e^{\phi/2} * G) - H\wedge G\,. \qquad (2.4)
\end{aligned}
$$

In addition, the forms obey the Bianchi identities

$$
\begin{aligned}
dB' &= H\\
dH &= 0\\
dG &= 2mB'\wedge H\,. \qquad (2.5)
\end{aligned}
$$

To make contact with the massless IIA supergravity of [26 – 28] one introduces a Stückelberg gauge potential A, with field strength $F = dA$, so that

$$mB' = mB + \frac{1}{2}F\,. \qquad (2.6)$$

In the massless limit, $m \longrightarrow 0$, we have $mB' \longrightarrow \frac{1}{2}F$.

Supersymmetry. The gravitino and dilatino supersymmetry variations read

$$\delta\Psi_M = \mathcal{D}_M\epsilon \qquad (2.7)$$

and

$$
\begin{aligned}
\delta\lambda = \Big\{ &-\frac{1}{2}\Gamma^M\nabla_M\phi - \frac{5m\ e^{5\phi/4}}{4} + \frac{3m\ e^{3\phi/4}}{8}B'_{MN}\Gamma^{MN}\Gamma_{11} + \\
&+ \frac{e^{-\phi/2}}{24}H_{MNP}\Gamma^{MNP}\Gamma_{11} - \frac{e^{\phi/4}}{192}G_{MNPQ}\Gamma^{MNPQ}\Big\}\epsilon\,, \qquad (2.8)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{D}_M := \Big\{ &\nabla_M - \frac{m\ e^{5\phi/4}}{16}\Gamma_M - \frac{m\ e^{3\phi/4}}{32}B'_{NP}(\Gamma_M{}^{NP} - 14\delta_M{}^N\Gamma^P)\Gamma_{11} + \frac{e^{-\phi/2}}{96}H_{NPQ}\times\\
&\times(\Gamma_M{}^{NPQ} - 9\delta_M{}^N\Gamma^{PQ})\Gamma_{11} + \frac{e^{\phi/4}}{256}G_{NPQR}\left(\Gamma_M{}^{NPQR} - \frac{20}{3}\delta_M{}^N\Gamma^{PQR}\right)\Big\}\,. \qquad (2.9)
\end{aligned}
$$

One can transform to the string frame by rescaling $e_A{}^M \to e^{\phi/4}e_A{}^M$.

JHEP08(2005)056

Integrability. It was shown in [3] that imposing supersymmetry together with the equations of motion for the forms implies the dilaton equation and the Einstein equation $E_{MN} = 0$, provided $E_{M0} = 0$ for $M \neq 0$.[5]

3. $\mathcal{N} = 1$ $M_{1,3} \times_\omega X_6$ backgrounds

Let us now assume that spacetime is of the form of a warped product $M_{1,3} \times_\omega X_6$, where $M_{1,3}$ is Minkowski or AdS_4 and X_6 is a compact manifold. The ten dimensional metric reads

$$g_{MN}(x, y) = \begin{pmatrix} \Delta^2(y)\hat{g}_{\mu\nu}(x) & 0 \\ 0 & \rho_{mn}(y) \end{pmatrix}, \tag{3.1}$$

where x is a coordinate on $M_{1,3}$ and y is a coordinate on X_6. We will also assume that the forms have nonzero y-dependent components along the internal directions, except for the four-form which will be allowed to have an additional component proportional to the volume of $M_{1,3}$

$$G_{\mu\nu\kappa\lambda} = \sqrt{g_4} f(y) \varepsilon_{\mu\nu\kappa\lambda}\,, \tag{3.2}$$

where f is a real scalar function on X_6. Note that with these assumptions the $E_{M0} = 0$ for $M \neq 0$ condition is satisfied, and therefore we need only check supersymmetry the Bianchi identities and the equations of motion for the forms.

3.1 Massive $\mathcal{N} = 1$ vacua and SU(2) structure

On $M_{1,3}$ there is a pair of Weyl spinors (related by complex conjugation), each of which satisfies the Killing equation

$$\hat{\nabla}_\mu \theta_+ = W \hat{\gamma}_\mu \theta_-\,; \qquad \hat{\nabla}_\mu \theta_- = W^* \hat{\gamma}_\mu \theta_+\,, \tag{3.3}$$

where hatted quantities are computed using the metric $\hat{g}_{\mu\nu}$, and the complex constant W is related to the scalar curvature $\hat{R}$ of $M_{1,3}$ through $\hat{R} = -24|W|^2$. The reader is referred to [3] for further details on our spinor conventions in four, six, ten dimensions.

It can been shown (see for example [18] for a recent discussion) that the requirement of $\mathcal{N} = 1$ supersymmetry[6] implies that the Majorana-Weyl supersymmetry parameter ϵ is decomposed under $Spin(1, 9) \longrightarrow Spin(1, 3) \times Spin(6)$ as

$$\epsilon = \alpha(y)\theta_+ \otimes \eta_{1+} + \delta(y)\theta_+ \otimes \eta_{2-} + \text{ c.c.}\,, \tag{3.4}$$

where α, δ, are complex functions on X_6 and $\eta_{1,2}$ is a pair of globally-defined, *nowhere-vanishing* Weyl spinors on X_6. Moreover, $\eta_{1,2-}$ are related to $\eta_{1,2+}$ by complex conjugation. Without loss of generality, we can choose $\eta_{1,2}$ to be of unit norm. In keeping with four-dimensional supersymmetry nomenclature, we take θ ($\eta_{1,2}$) to be anticommuting (commuting).

[5] Similar integrability conditions were derived in [29 – 31] in the context of eleven-dimensional supergravity. See also [32] for a recent general discussion.

[6] This corresponds to four real supercharges; in the present paper we are counting supersymmetries according to four-dimensional conventions.

There are three cases according to the relation between η_1 and η_2: *a)* η_2 is everywhere parallel to η_1 and X_6 is of SU(3) structure, *b)* η_2 is nowhere parallel to η_1 and X_6 is of SU(2) structure, or *c)* at generic points in X_6 $\eta_{1,2}$ are linearly independent, but there exist points where η_2 becomes parallel to η_1. The latter case imposes no additional topological requirement on X_6 other than that its structure group should reduce to SU(3).

In the present paper we will take η_2 to be everywhere *orthogonal* to η_1: $\eta_{2+}^{+}\eta_{1+}=0$. This is a special sub-case of b) above and therefore X_6 must be a manifold of SU(2) structure. As we will see later in section 5, requiring in addition that the Romans' mass be nonzero implies that up to a choice of phase which can be absorbed in the normalization of the spinors $\eta_{1,2}$,

$$|\alpha|=\alpha=\delta\,. \tag{3.5}$$

3.2 Reduction of the supersymmetry conditions

Substituting the spinor Ansatz (3.4) in the supersymmetry transformations we obtain

$$\begin{aligned}
0 =\ & \alpha\nabla_m\eta_{1+}+\partial_m\alpha\eta_{1+}+\alpha\frac{e^{-\phi/2}}{96}H_{npq}(\gamma_m{}^{npq}-9\delta_m{}^n\gamma^{pq})\eta_{1+}-\delta\frac{me^{5\phi/4}}{16}\gamma_m\eta_{2-}+\\
&+3i\delta f\frac{e^{\phi/4}}{32}\gamma_m\eta_{2-}+\delta\frac{me^{3\phi/4}}{32}B'_{np}(\gamma_m{}^{np}-14\delta_m{}^n\gamma^p)\eta_{2-}+\\
&+\delta\frac{e^{\phi/4}}{256}G_{npqr}\left(\gamma_m{}^{npqr}-\frac{20}{3}\delta_m{}^n\gamma^{pqr}\right)\eta_{2-}
\end{aligned}\tag{3.6}$$

$$\begin{aligned}
0 =\ & \delta^*\nabla_m\eta_{2+}+\partial_m\delta^*\eta_{2+}-\delta^*\frac{e^{-\phi/2}}{96}H_{npq}(\gamma_m{}^{npq}-9\delta_m{}^n\gamma^{pq})\eta_{2+}+\alpha^*\frac{me^{5\phi/4}}{16}\gamma_m\eta_{1-}+\\
&+3i\alpha^* f\frac{e^{\phi/4}}{32}\gamma_m\eta_{1-}+\alpha^*\frac{me^{3\phi/4}}{32}B'_{np}(\gamma_m{}^{np}-14\delta_m{}^n\gamma^p)\eta_{1-}-\\
&-\alpha^*\frac{e^{\phi/4}}{256}G_{npqr}\left(\gamma_m{}^{npqr}-\frac{20}{3}\delta_m{}^n\gamma^{pqr}\right)\eta_{1-}\,,
\end{aligned}\tag{3.7}$$

from the 'internal' components of the gravitino variation and

$$\begin{aligned}
0 =\ & \alpha\Delta^{-1}W\eta_{1+}+\delta^*\frac{me^{5\phi/4}}{16}\eta_{2+}-5i\delta^* f\frac{e^{\phi/4}}{32}\eta_{2+}-\delta^*\frac{me^{3\phi/4}}{32}B'_{mn}\gamma^{mn}\eta_{2+}+\\
&+\alpha^*\frac{e^{-\phi/2}}{96}H_{mnp}\gamma^{mnp}\eta_{1-}-\delta^*\frac{e^{\phi/4}}{256}G_{mnpq}\gamma^{mnpq}\eta_{2+}-\frac{1}{2}\alpha^*\partial_m(\ln\Delta)\gamma^m\eta_{1-}
\end{aligned}\tag{3.8}$$

$$\begin{aligned}
0 =\ & \delta^*\Delta^{-1}W^*\eta_{2+}+\alpha\frac{me^{5\phi/4}}{16}\eta_{1+}+5i\alpha f\frac{e^{\phi/4}}{32}\eta_{1+}+\alpha\frac{me^{3\phi/4}}{32}B'_{mn}\gamma^{mn}\eta_{1+}+\\
&+\delta\frac{e^{-\phi/2}}{96}H_{mnp}\gamma^{mnp}\eta_{2-}-\alpha\frac{e^{\phi/4}}{256}G_{mnpq}\gamma^{mnpq}\eta_{1+}+\frac{1}{2}\delta\partial_m(\ln\Delta)\gamma^m\eta_{2-}\,,
\end{aligned}\tag{3.9}$$

from the noncompact piece. Note that these equations are complex. Similarly from the dilatino we obtain

$$\begin{aligned}
0 =\ & \frac{1}{2}\alpha^*\partial_m\phi\gamma^m\eta_{1-}-\alpha^*\frac{e^{-\phi/2}}{24}H_{mnp}\gamma^{mnp}\eta_{1-}-\delta^*\frac{5me^{5\phi/4}}{4}\eta_{2+}+\\
&+i\delta^* f\frac{e^{\phi/4}}{8}\eta_{2+}-\delta^*\frac{3me^{3\phi/4}}{8}B'_{mn}\gamma^{mn}\eta_{2+}-\delta^*\frac{e^{\phi/4}}{192}G_{mnpq}\gamma^{mnpq}\eta_{2+}
\end{aligned}\tag{3.10}$$

JHEP08(2005)056

$$0 = \frac{1}{2}\delta\partial_m\phi\gamma^m\eta_{2-} + \delta\frac{e^{-\phi/2}}{24}H_{mnp}\gamma^{mnp}\eta_{2-} + \alpha\frac{5me^{5\phi/4}}{4}\eta_{1+} + + i\alpha f\frac{e^{\phi/4}}{8}\eta_{1+} - \alpha\frac{3me^{3\phi/4}}{8}B'_{mn}\gamma^{mn}\eta_{1+} + \alpha\frac{e^{\phi/4}}{192}G_{mnpq}\gamma^{mnpq}\eta_{1+}\,. \tag{3.11}$$

4. SU(2) reduction

The analysis of the conditions for a supersymmetric vacuum and the characterization of the solutions is greatly facilitated by using the machinery of G-structures [1]. The existence of two globally-defined nowhere-vanishing orthogonal spinors $\eta_{1,2}$, as is the case here, implies the reduction of the structure group of X_6 to SU(2). This allows us to decompose all tensors on X_6 in terms of irreducible SU(2) representations. In the following two subsections we review some of the relevant facts about SU(3) and SU(2) structures, before we turn to the analysis of the conditions for an $\mathcal{N} = 1$ supersymmetric vacuum in section 5. The details of the SU(2) decomposition of the antisymmetric forms of IIA supergravity and the SU(2) decomposition of the supersymmetry conditions are given in appendices C, D respectively.

4.1 SU(3) structure

The existence of a nowhere-vanishing globally-defined spinor η_1 allows us to define the bilinears

$$J_{mn} := i\eta^+_{1-}\gamma_{mn}\eta_{1-} = -i\eta^+_{1+}\gamma_{mn}\eta_{1+} \tag{4.1}$$

$$\Omega_{mnp} := \eta^+_{1-}\gamma_{mnp}\eta_{1+}\,; \qquad \Omega^*_{mnp} = -\eta^+_{1+}\gamma_{mnp}\eta_{1-}\,. \tag{4.2}$$

Note that J_{mn} thus defined is real and Ω (Ω^*) is imaginary (anti-) self-dual.

$$\Omega_{mnp} = \frac{i}{6}\sqrt{\rho_6}\;\varepsilon_{mnpijk}\Omega^{ijk}\,. \tag{4.3}$$

We choose to normalize

$$\eta^+_{1+}\eta_{1+} = \eta^+_{1-}\eta_{1-} = 1\,. \tag{4.4}$$

Using (A.1) one can prove that J, Ω satisfy

$$J_m{}^n J_n{}^p = -\delta_m{}^p \tag{4.5}$$

$$(\Pi^+)_m{}^n\Omega_{npq} = \Omega_{mpq}\,; \qquad (\Pi^-)_m{}^n\Omega_{npq} = 0\,, \tag{4.6}$$

where

$$(\Pi^\pm)_m{}^n := \frac{1}{2}\left(\delta_m{}^n \mp iJ_m{}^n\right) \tag{4.7}$$

are the projection operators onto the holomorphic/antiholomorphic parts. In other words, J defines an almost complex structure with respect to which Ω is $(3,0)$. Moreover (using (A.1) again) it follows that

$$\begin{aligned}\Omega\wedge J &= 0\\ \Omega\wedge\Omega^* &= \frac{4i}{3}J^3\,.\end{aligned} \tag{4.8}$$

Therefore J, Ω, completely specify an SU(3) structure on X_6.

Further useful relations can be found in [3].

4.2 SU(2) structure

The existence of two orthogonal unimodular globally-defined spinors η_1, η_2 on X_6 allows us to define two distinct SU(3) structures

$$J_{mn} := i\eta^+_{1-}\gamma_{mn}\eta_{1-}\,; \qquad \Omega_{mnp} := \eta^+_{1-}\gamma_{mnp}\eta_{1+} \tag{4.9}$$

and

$$J'_{mn} := i\eta^+_{2-}\gamma_{mn}\eta_{2-}\,; \qquad \Omega'_{mnp} := \eta^+_{2-}\gamma_{mnp}\eta_{2+}\,. \tag{4.10}$$

Each of these satisfies all the properties of SU(3) structures given in the preceding section. It can be shown, using the above definitions and the Fierz identities in appendix A, that

$$\begin{aligned} J &= -\frac{i}{2}K\wedge K^* + \widetilde{J} \\ J' &= -\frac{i}{2}K\wedge K^* - \widetilde{J}\,, \end{aligned} \tag{4.11}$$

where

$$K_m := \eta^+_{2-}\gamma_m\eta_{1+} \tag{4.12}$$

and

$$\iota_K\widetilde{J} = 0\,. \tag{4.13}$$

The complex vector K satisfies

$$K_mK^m = 0\,; \qquad K^*_mK^m = 2 \tag{4.14}$$

and is holomorphic with respect to J,

$$(\Pi^+)_m{}^nK_n = K_m\,; \qquad (\Pi^-)_m{}^nK_n = 0\,. \tag{4.15}$$

It follows that the two-form $\widetilde{J}$ is $(1,1)$ with respect to the almost complex structure J.

The two holomorphic three-forms can be expressed in the following way

$$\begin{aligned} \Omega &= -iK\wedge\omega \\ \Omega' &= iK\wedge\omega^*\,, \end{aligned} \tag{4.16}$$

where

$$\omega_{mn} := i\eta^+_{1-}\gamma_{mn}\eta_{2-} \tag{4.17}$$

satisfies

$$\iota_K\omega = \iota_{K^*}\omega = 0 \tag{4.18}$$

and is holomorphic with respect to J,

$$(\Pi^+)_m{}^n\omega_{np} = \omega_{mp}\,; \qquad (\Pi^-)_m{}^n\omega_{np} = 0\,. \tag{4.19}$$

It is straightforward to show that $\widetilde{J}$, ω specify an SU(2) structure. Indeed, it follows from the above formulæ that

$$\begin{aligned} \widetilde{J}\wedge\omega &= 0 \\ \omega\wedge\omega^* &= 2\widetilde{J}\wedge\widetilde{J}\,. \end{aligned} \tag{4.20}$$

The complex vector K specifies an almost product structure

$$R_m{}^n := K_m K^{*n} + K^*_m K^n - \delta_m{}^n \,, \tag{4.21}$$

such that

$$R_m{}^n R_n{}^p = \delta_m{}^p \,. \tag{4.22}$$

Further useful relations are given in appendix B.

5. Analysis of the conditions

To analyze the supersymmetry conditions of section 3.2 it is useful to note that equations (3.6), (3.7) can be cast in the form

$$U_m \eta_{1+} + U_{mn} \gamma^n \eta_{1-} = 0 \,, \tag{5.1}$$

whereas equations (3.8)–(3.11) can be written as

$$V \eta_{1+} + V_m \gamma^m \eta_{1-} = 0 \,. \tag{5.2}$$

The explicit expressions for the U's and V's can be read off from the expressions in appendix D. The tensors U, V further decompose into directions parallel and perpendicular to the complex vector K defined in 4.2. The components perpendicular to K are further decomposed in terms of irreducible SU(2) representations. The details of the decomposition are given in appendix C. To illustrate the procedure, let us decompose

$$V_m = v_m + v K_m \,, \tag{5.3}$$

where $K^m v_m = K^{*m} v_m = 0$. We also noted that in the decomposition of V_m there are no terms proportional to K^*_m, due to A.7. It follows that the scalar content of (5.2) is equivalent to:

$$V = v = 0 \,. \tag{5.4}$$

We proceed similarly for all other representations.

Let us consider the scalar component of the supersymmetry equations first. It is straightforward to show that if there exists a point y_0 in X_6 such that $|\alpha(y_0)| \neq |\delta(y_0)|$, equations (3.8)–(3.11) imply that $m = W = 0$. I.e. the mass parameter vanishes and the space $M_{1,3}$ reduces to Minkowski. We would like to look for massive solutions of IIA and hence, up to phases which can be absorbed in the normalizations of $\eta_{1,2}$ we can take:

$$|\alpha| = \alpha = \delta \,, \tag{5.5}$$

at each point in X_6. Let us first analyze the supersymmetry equations (3.8)–(3.11), considering each irreducible SU(2) representation in turn. The decompositions of all antisymmetric tensors in terms of irreducible SU(2) representations can be found in appendix C. One can show that the solution is equivalent to the following conditions:

The 1.

$$\begin{aligned} f &= 0 \\ W &= \frac{i\Delta}{8}\left(me^{3\phi/4}b_2^* + \frac{i}{12}e^{\phi/4}g_2^*\right), \end{aligned} \tag{5.6}$$

$$\begin{aligned} mb_1 &= 0 \\ me^{3\phi/4}b_3 &= \frac{i}{2}(d_K\phi_- - d_{K^*}\phi_-)\,, \end{aligned} \tag{5.7}$$

where $d_K := K^m\partial_m$, $d_{K^*} := K^{*m}\partial_m$ and $\phi_\pm := \phi \pm 4\ln\Delta$. Also

$$\begin{aligned} e^{\phi/4}g_1 &= 16\left(me^{5\phi/4} + \frac{1}{4}d_K\phi_- + \frac{1}{4}d_{K^*}\phi_-\right) \\ g_3 &= 0\,, \end{aligned} \tag{5.8}$$

$$\begin{aligned} e^{-\phi/2}h_1 &= e^{-\phi/2}h_2^* = \frac{9me^{3\phi/4}}{2}b_2 - \frac{ie^{\phi/4}}{8}g_2 \\ e^{-\phi/2}h_3 &= 12i\left(me^{5\phi/4} + \frac{1}{8}d_K\phi_- + \frac{1}{4}d_{K^*}\phi_+\right). \end{aligned} \tag{5.9}$$

The 2.

$$\widetilde{\partial}^+_m \ln\Delta = \frac{1}{4}\widetilde{\partial}^+_m\phi - \frac{me^{3\phi/4}}{8}\omega_m{}^n\widetilde{b}^*_{1n} - \frac{e^{\phi/4}}{64}\widetilde{g}^*_{2m}\,, \tag{5.10}$$

where $\widetilde{\partial}_m$ is defined in (D.15) and $\widetilde{\partial}^\pm_m := (\widetilde{\Pi}^\pm)_m{}^n\widetilde{\partial}_n$. Moreover

$$me^{3\phi/4}\widetilde{b}_{2m} = -me^{3\phi/4}\widetilde{b}^*_{1m} - \frac{e^{\phi/4}}{32}\omega^*_m{}^n(\widetilde{g}_{1n} - \widetilde{g}^*_{2n})\,, \tag{5.11}$$

$$\begin{aligned} e^{-\phi/2}\widetilde{h}_{1m} &= \frac{e^{\phi/4}}{4}(\widetilde{g}_{1m} - \widetilde{g}^*_{2m}) \\ e^{-\phi/2}\widetilde{h}_{2m} &= -3i\left\{2\omega_m{}^n\widetilde{\partial}^-_n\phi + 3me^{3\phi/4}\widetilde{b}_{1m} + \frac{e^{\phi/4}}{16}\omega_m{}^n\left(\frac{1}{2}\widetilde{g}^*_{1n} - \widetilde{g}_{2n}\right)\right\}, \end{aligned} \tag{5.12}$$

The 3. This representation drops out of equations (3.8)–(3.11).

Next we turn to the equations (3.6), (3.7). The fact that $\eta_{1,2}$ are unimodular implies $\nabla(\eta_1^+\eta_1) = 0$ and $\nabla(\eta_2^+\eta_2) = 0$ which, taking (3.6), (3.7) into account, can be seen to be equivalent to

$$\alpha = \text{constant} \times \Delta^{-1/2}\,. \tag{5.13}$$

In addition, the orthogonality of $\eta_{1,2}$ implies $\nabla(\eta_1^+\eta_2) = 0$ which, taking (3.6), (3.7) into account, leads to the condition

$$h_1 = h_2^* = \frac{ie^{3\phi/4}}{4}g_2\,. \tag{5.14}$$

Comparing with (5.9), (5.6) we conclude that $mb_2 = ie^{-\phi/2}g_2/12$ and $W = 0$. Note that the **2** representation drops out of the orthogonality constraints.

To summarize the conditions so far:

$$f\,,\ W = 0\,. \tag{5.15}$$

In addition, in form notation,

$$\begin{aligned}
mB' &= \left[\frac{im}{4}\mathrm{Im}(b_1') + \frac{1}{64}e^{-\phi/2}(\widetilde{g}_1 - \widetilde{g}_2^*)\right]\wedge K + \text{c.c.} + \\
&\quad + m\widetilde{b} - \frac{e^{-3\phi/4}}{4}\widetilde{J}\,\mathrm{Im}(d_K\phi_-) + \frac{e^{-\phi/2}}{48}\mathrm{Im}(\omega g_2^*) \\
H &= \frac{1}{3}\left\{\widetilde{h} + \frac{e^{3\phi/4}}{16}\mathrm{Im}(\omega g_2^*) + 3ie^{\phi/2}\widetilde{J}\left[me^{5\phi/4} + \frac{1}{8}d_K\phi_- + \frac{1}{4}d_{K^*}\phi_+\right]\right\}\wedge K + \text{c.c.} - \\
&\quad - \frac{e^{3\phi/4}}{16}\widetilde{J}\wedge\mathrm{Im}(\widetilde{g}_1 + \widetilde{g}_2) + 2i\mathrm{Im}\left\{e^{\phi/2}d^+\phi + \frac{3e^{5\phi/4}}{8}b_1' + \frac{e^{3\phi/4}}{32}\left(\frac{1}{2}\widetilde{g}_1 + \widetilde{g}_2\right)\right\}\wedge K\wedge K^* \\
G &= \left[me^{\phi} + \frac{e^{-\phi/4}}{2}\mathrm{Re}(d_K\phi_-)\right]\widetilde{J}\wedge\widetilde{J} - \\
&\quad - \frac{i}{32}\left\{(\widetilde{g}_1 - \widetilde{g}_2)\wedge\widetilde{J}\wedge K + \text{c.c.}\right\} - \frac{i}{12}\left[\widetilde{g} + \frac{1}{4}\mathrm{Re}(\omega g_2^*)\right]\wedge K\wedge K^*\,,
\end{aligned} \tag{5.16}$$

where we have defined $b'_{1m} := \omega^*_m{}^n b_{1n}$. The differential equations (3.6), (3.7) determine the specific SU(2) structure of X_6 and impose further constraints. In order to search for explicit solutions we will now make some simplifying assumptions. Namely, we will assume that only scalar fluxes are present (i.e. only the **1** components of the form fields are nonzero) and that the dilaton is constant.[7] We use (3.6), (3.7) to read off the exterior derivatives on Ω, J and K:

$$\begin{aligned}
d\Omega &= -i\left(m\omega + \frac{g_2}{48}\widetilde{J}\right)\wedge K\wedge K^* + \frac{g_2}{12}\widetilde{J}\wedge\widetilde{J} \\
dJ &= -\frac{1}{24}\mathrm{Re}(\omega g_2^*)\wedge\mathrm{Re}(K) \\
dK &= mK\wedge K^* + \frac{i}{24}\mathrm{Re}(\omega g_2^*)\,.
\end{aligned} \tag{5.17}$$

Moreover, we can read off the action of the exterior derivative on the SU(2) structure

$$\begin{aligned}
d\widetilde{J} &= 0 \\
d\omega &= \frac{g_2}{48}\widetilde{J}\wedge K^*\,.
\end{aligned} \tag{5.18}$$

Combining all the above, we note that the nilpotency of the exterior derivative $d^2 = 0$ implies

$$g_2 = 0\,. \tag{5.19}$$

It is now straightforward to see that the Bianchi identities imply that all fluxes are zero and $m = 0$, contrary to our assumption. We therefore conclude that there are no solutions obeying our simplified Ansatz.

[7] In the remainder of this section we will absorb all dilaton dependence by a field redefinition of the forms.

6. $\mathcal{N}=2$ AdS_4 vacua and SU(2) structure

In this section we will search for $\mathcal{N}=2$ supersymmetric vacua of the type $AdS_4 \times_\omega X_6$. In order to simplify the computation, we will not consider the most general spinor Ansatz. Instead we demand that the background be invariant under two supersymmetries $\epsilon_{1,2}$ of the form

$$\epsilon_1 = \alpha(y)\theta_+ \otimes \eta_{1+} + \beta(y)\theta_+ \otimes \eta_{1-} + \text{c.c.}\,, \tag{6.1}$$

and

$$\epsilon_2 = \gamma(y)\theta_+ \otimes \eta_{2+} + \delta(y)\theta_+ \otimes \eta_{2-} + \text{ c.c.}\,, \tag{6.2}$$

where α, β, γ, δ, are complex functions on X_6 and $\eta_{1,2}$ are globally-defined, unimodular spinors on X_6. In addition we will take $\eta_{1,2}$ to be orthogonal to each other. Consequently, X_6 must be a manifold of SU(2) structure. As we will see later in section 6.2, under these assumptions supersymmetry implies that up to a choice of phase which can be absorbed in the normalizations of the spinors $\eta_{1,2}$,

$$\begin{aligned} \alpha &= \beta \\ \gamma &= \delta\,. \end{aligned} \tag{6.3}$$

6.1 Reduction of the supersymmetry conditions

Substituting the spinor Ansatz (6.1) in the supersymmetry transformations we obtain

$$\begin{aligned} 0 =\ & \alpha\nabla_m\eta_{1+} + \partial_m\alpha\eta_{1+} + \alpha\frac{e^{-\phi/2}}{96}H_{npq}(\gamma_m{}^{npq} - 9\delta_m{}^n\gamma^{pq})\eta_{1+} - \beta\frac{me^{5\phi/4}}{16}\gamma_m\eta_{1-} + \\ & + 3i\beta f\frac{e^{\phi/4}}{32}\gamma_m\eta_{1-} + \beta\frac{me^{3\phi/4}}{32}B'_{np}(\gamma_m{}^{np} - 14\delta_m{}^n\gamma^p)\eta_{1-} + \\ & + \beta\frac{e^{\phi/4}}{256}G_{npqr}\left(\gamma_m{}^{npqr} - \frac{20}{3}\delta_m{}^n\gamma^{pqr}\right)\eta_{1-} \end{aligned} \tag{6.4}$$

$$\begin{aligned} 0 =\ & \beta^*\nabla_m\eta_{1+} + \partial_m\beta^*\eta_{1+} - \beta^*\frac{e^{-\phi/2}}{96}H_{npq}(\gamma_m{}^{npq} - 9\delta_m{}^n\gamma^{pq})\eta_{1+} + \alpha^*\frac{me^{5\phi/4}}{16}\gamma_m\eta_{1-} + \\ & + 3i\alpha^* f\frac{e^{\phi/4}}{32}\gamma_m\eta_{1-} + \alpha^*\frac{me^{3\phi/4}}{32}B'_{np}(\gamma_m{}^{np} - 14\delta_m{}^n\gamma^p)\eta_{1-} - \\ & - \alpha^*\frac{e^{\phi/4}}{256}G_{npqr}\left(\gamma_m{}^{npqr} - \frac{20}{3}\delta_m{}^n\gamma^{pqr}\right)\eta_{1-}\,, \end{aligned} \tag{6.5}$$

from the 'internal' components of the gravitino variation and

$$\begin{aligned} 0 =\ & \alpha\Delta^{-1}W\eta_{1+} + \beta^*\frac{me^{5\phi/4}}{16}\eta_{1+} - 5i\beta^* f\frac{e^{\phi/4}}{32}\eta_{1+} - \beta^*\frac{me^{3\phi/4}}{32}B'_{mn}\gamma^{mn}\eta_{1+} + \\ & + \alpha^*\frac{e^{-\phi/2}}{96}H_{mnp}\gamma^{mnp}\eta_{1-} - \beta^*\frac{e^{\phi/4}}{256}G_{mnpq}\gamma^{mnpq}\eta_{1+} - \frac{1}{2}\alpha^*\partial_m(\ln\Delta)\gamma^m\eta_{1-} \end{aligned} \tag{6.6}$$

$$\begin{aligned} 0 =\ & \beta^*\Delta^{-1}W^*\eta_{1+} + \alpha\frac{me^{5\phi/4}}{16}\eta_{1+} + 5i\alpha f\frac{e^{\phi/4}}{32}\eta_{1+} + \alpha\frac{me^{3\phi/4}}{32}B'_{mn}\gamma^{mn}\eta_{1+} + \\ & + \beta\frac{e^{-\phi/2}}{96}H_{mnp}\gamma^{mnp}\eta_{1-} - \alpha\frac{e^{\phi/4}}{256}G_{mnpq}\gamma^{mnpq}\eta_{1+} + \frac{1}{2}\beta\partial_m(\ln\Delta)\gamma^m\eta_{1-}\,, \end{aligned} \tag{6.7}$$

from the noncompact piece. Note that these equations are complex. Similarly from the dilatino we obtain

$$\begin{aligned}
0 &= \frac{1}{2}\alpha^*\partial_m\phi\gamma^m\eta_{1-} - \alpha^*\frac{e^{-\phi/2}}{24}H_{mnp}\gamma^{mnp}\eta_{1-} - \beta^*\frac{5me^{5\phi/4}}{4}\eta_{1+} + \\
&\quad + i\beta^* f\frac{e^{\phi/4}}{8}\eta_{1+} - \beta^*\frac{3me^{3\phi/4}}{8}B'_{mn}\gamma^{mn}\eta_{1+} - \beta^*\frac{e^{\phi/4}}{192}G_{mnpq}\gamma^{mnpq}\eta_{1+} \qquad (6.8)\\
0 &= \frac{1}{2}\beta\partial_m\phi\gamma^m\eta_{1-} + \beta\frac{e^{-\phi/2}}{24}H_{mnp}\gamma^{mnp}\eta_{1-} + \alpha\frac{5me^{5\phi/4}}{4}\eta_{1+} + \\
&\quad + i\alpha f\frac{e^{\phi/4}}{8}\eta_{1+} - \alpha\frac{3me^{3\phi/4}}{8}B'_{mn}\gamma^{mn}\eta_{1+} + \alpha\frac{e^{\phi/4}}{192}G_{mnpq}\gamma^{mnpq}\eta_{1+}\,. \qquad (6.9)
\end{aligned}$$

A second set of conditions follows from the second supersymmetry (6.2). These can be obtained from the ones above by substituting $(\alpha,\beta,\eta_1)\longrightarrow(\gamma,\delta,\eta_2)$.

6.2 Analysis of the conditions

Let us consider the scalar component of the supersymmetry equations first. It is straightforward to show that if there exists a point y_0 in X_6 such that $|\alpha(y_0)|\neq|\beta(y_0)|$ or $|\gamma(y_0)|\neq|\delta(y_0)|$, equations (6.6)–(6.9) (and the ones obtained from them by substituting $(\alpha,\beta,\eta_1)\longrightarrow(\gamma,\delta,\eta_2)$) imply that $m,\ f,\ W=0$. I.e. the space $M_{1,3}$ reduces to Minkowski, which is contrary to our assumption. Hence, up to phases which can be absorbed in the normalizations of $\eta_{1,2}$ we can take:

$$\begin{aligned}
\alpha &= \beta \\
\gamma &= \delta\,, \qquad (6.10)
\end{aligned}$$

at each point in X_6. Taking (6.10) into account, it is useful to note that the supersymmetry conditions (6.4), (6.5), as well as the ones obtained from them by substituting $(\alpha,\eta_1)\longrightarrow(\gamma,\eta_2)$, are equivalent to the following set of equations:

$$0 = \nabla_m\eta_{1+} + \partial_m\ln|\alpha|\eta_{1+} + 3if\frac{e^{\phi/4}}{32}\gamma_m\eta_{1-} + \frac{me^{3\phi/4}}{32}B'_{np}({\gamma_m}^{np} - 14{\delta_m}^n\gamma^p)\eta_{1-} \qquad (6.11)$$

$$\begin{aligned}
0 &= \partial_m\ln\Big(\frac{\alpha}{|\alpha|}\Big)\eta_{1+} + \frac{e^{-\phi/2}}{96}H_{npq}({\gamma_m}^{npq} - 9{\delta_m}^n\gamma^{pq})\eta_{1+} - \frac{me^{5\phi/4}}{16}\gamma_m\eta_{1-} + \\
&\quad + \frac{e^{\phi/4}}{256}G_{npqr}\left({\gamma_m}^{npqr} - \frac{20}{3}{\delta_m}^n\gamma^{pqr}\right)\eta_{1-} \qquad (6.12)
\end{aligned}$$

and

$$0 = \nabla_m\eta_{2+} + \partial_m\ln|\gamma|\eta_{2+} + 3if\frac{e^{\phi/4}}{32}\gamma_m\eta_{2-} + \frac{me^{3\phi/4}}{32}B'_{np}({\gamma_m}^{np} - 14{\delta_m}^n\gamma^p)\eta_{2-} \qquad (6.13)$$

$$\begin{aligned}
0 &= \partial_m\ln\Big(\frac{\gamma}{|\gamma|}\Big)\eta_{2+} + \frac{e^{-\phi/2}}{96}H_{npq}({\gamma_m}^{npq} - 9{\delta_m}^n\gamma^{pq})\eta_{2+} - \frac{me^{5\phi/4}}{16}\gamma_m\eta_{2-} + \\
&\quad + \frac{e^{\phi/4}}{256}G_{npqr}\left({\gamma_m}^{npqr} - \frac{20}{3}{\delta_m}^n\gamma^{pqr}\right)\eta_{2-}\,. \qquad (6.14)
\end{aligned}$$

Let us first analyze the supersymmetry equations (6.12), (6.14), (6.6)–(6.9) and the ones obtained from them by $(\alpha, \eta_1) \longrightarrow (\gamma, \eta_2)$, considering each irreducible SU(2) representation in turn. The decompositions of all antisymmetric tensors in terms of irreducible SU(2) representations can be found in appendix C. One can show that the solution is equivalent to the following conditions:

The 1.

$$\begin{aligned} m &= 0 \\ W &= \frac{i\Delta}{6}\left(\frac{\alpha}{|\alpha|}\right)^{-2} f e^{\phi/4} . \end{aligned} \tag{6.15}$$

$$\begin{aligned} mb_1 &= \frac{f}{6} e^{-\phi/2} \\ mb_2 &= \frac{4i}{3} e^{-3\phi/4} d_K \phi \\ mb_3 &= 0 , \end{aligned} \tag{6.16}$$

$$g_1 = g_2 = g_3 = 0 , \tag{6.17}$$

$$h_1 = h_2 = h_3 = 0 , \tag{6.18}$$

$$\begin{aligned} \frac{\alpha}{|\alpha|} &= \pm \frac{\gamma}{|\gamma|} \\ d_K\left(\frac{\alpha}{|\alpha|}\right) &= d_{K^*}\left(\frac{\alpha}{|\alpha|}\right) = 0 , \end{aligned} \tag{6.19}$$

$$\begin{aligned} d_K \ln\Delta &= -\frac{1}{12} d_K \phi \\ d_K \phi &= d_{K^*} \phi . \end{aligned} \tag{6.20}$$

The first line of (6.20) together with the second line of (6.15) and the fact that W is constant, imply:

$$d_K \ln f = -\frac{1}{6} d_K \phi . \tag{6.21}$$

The 2.

$$\begin{aligned} m\widetilde{b}_{1m} &= \frac{4i}{3} e^{-3\phi/4} \widetilde{\partial}^+_m \phi \\ m\widetilde{b}_{2m} &= -\frac{4i}{3} e^{-3\phi/4} \widetilde{\partial}^-_m \phi , \end{aligned} \tag{6.22}$$

$$\widetilde{h}_{1m} = \widetilde{h}_{2m} = 0 , \tag{6.23}$$

$$\begin{aligned} \widetilde{g}_{1m} &= \widetilde{g}_{2m} = 0 , \widetilde{\partial}_m\left(\frac{\alpha}{|\alpha|}\right) = 0 \\ \widetilde{\partial}_m \ln\Delta &= -\frac{1}{12} \widetilde{\partial}_m \phi . \end{aligned} \tag{6.24}$$

The 3.

$$\widetilde{h}_{mn} = \widetilde{g}_{mn} = 0 . \tag{6.25}$$

The relations derived so far imply $\Delta = \text{constant} \times e^{-\phi/12}$, $f = \text{constant} \times e^{-\phi/6}$, as well as $H = 0$, $G = f dVol_4$, where $dVol_4$ is the volume element of $M_{1,3}$ in the warped metric. It then follows from the Bianchi identity (2.5) for the G field that $\phi = \text{constant}$.

To summarize the conditions so far:

$$
\begin{aligned}
m &= 0 \\
W &= \frac{i\Delta}{6}\left(\frac{\alpha}{|\alpha|}\right)^{-2} f e^{\phi/4} \\
\frac{\gamma}{|\gamma|} &= \pm\frac{\alpha}{|\alpha|} \\
\frac{\alpha}{|\alpha|}\,,\ \Delta\,,\ \phi\,,\ f &= \text{constant}\,.
\end{aligned}
\tag{6.26}
$$

In addition, in form notation,

$$
\begin{aligned}
F &= \widetilde{f} - \frac{i}{6} f e^{-\phi/2} K \wedge K^* \\
H &= 0 \\
G &= f dVol_4\,.
\end{aligned}
\tag{6.27}
$$

Note that we have taken (2.6) and the fact that $m = 0$ into account, and we have set $m\widetilde{b}_{mn} = \frac{1}{2}\widetilde{f}_{mn}$.

Next we turn to the equations (6.11), (6.13). The fact that $\eta_{1,2}$ are unimodular implies $\nabla(\eta_1^+\eta_1) = 0$ and $\nabla(\eta_2^+\eta_2) = 0$ which, taking (6.11), (6.13) into account, can be seen to be equivalent to $|\alpha|,\ |\gamma| = \text{constant}$. Together with (6.26) this implies

$$
\alpha,\ \gamma = \text{constant}\,. \tag{6.28}
$$

In addition, the orthogonality of $\eta_{1,2}$ implies $\nabla(\eta_1^+\eta_2) = 0$ which, taking (6.11), (6.13) into account, leads to the condition

$$
f = 0\,. \tag{6.29}
$$

Taking (6.26) into account, this implies $W = 0$ and $M_{1,3}$ reduces to Minkowski space.[8] Of course, this is contrary to our assumption of a (warped) AdS_4 vacuum. We are therefore led to the conclusion that there are no $\mathcal{N} = 2$ solutions of type IIA supergravity satisfying our requirements.

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A. Fierz identities

Using definitions (4.1), (4.2) we find

$$
\eta_{1-}^{\alpha}\eta_{1+}^{\beta} = \frac{1}{4}(P_- C^{-1})^{\alpha\beta} + \frac{i}{8} J_{mn}(P_-\gamma^{mn}C^{-1})^{\alpha\beta}
$$

[8] It is not difficult to see that in addition the equations of motion impose $\widetilde{f} = 0$ and therefore all fluxes are zero.

JHEP08(2005)056

$$\begin{aligned}
\eta_{1+}^{\alpha}\eta_{1+}^{\beta} &= -\frac{1}{48}\Omega_{mnp}(P_+\gamma^{mnp}C^{-1})^{\alpha\beta} \\
\eta_{1-}^{\alpha}\eta_{1-}^{\beta} &= \frac{1}{48}\Omega^*_{mnp}(P_-\gamma^{mnp}C^{-1})^{\alpha\beta}
\end{aligned} \tag{A.1}$$

and similarly for $\eta_2 \otimes \eta_2$, by replacing $(J,\Omega) \to (J',\Omega')$. Moreover for $\eta_1 \otimes \eta_2$ we have

$$\begin{aligned}
\eta_{1-}^{\alpha}\eta_{2+}^{\beta} &= \frac{i}{8}\omega^*_{mn}(P_-\gamma^{mn}C^{-1})^{\alpha\beta} \\
\eta_{2-}^{\alpha}\eta_{1+}^{\beta} &= \frac{i}{8}\omega_{mn}(P_-\gamma^{mn}C^{-1})^{\alpha\beta} \\
\eta_{1+}^{\alpha}\eta_{2+}^{\beta} &= \frac{1}{4}K_m(P_+\gamma^{m}C^{-1})^{\alpha\beta} - \frac{1}{48}\widetilde{\Omega}_{mnp}(P_+\gamma^{mnp}C^{-1})^{\alpha\beta} \\
\eta_{1-}^{\alpha}\eta_{2-}^{\beta} &= -\frac{1}{4}K^*_m(P_-\gamma^{m}C^{-1})^{\alpha\beta} + \frac{1}{48}\widetilde{\Omega}^*_{mnp}(P_-\gamma^{mnp}C^{-1})^{\alpha\beta} \,,
\end{aligned} \tag{A.2}$$

where

$$\widetilde{\Omega}_{mnp} := (\eta_{2-}^{+}\gamma_{mnp}\eta_{1+}) \,. \tag{A.3}$$

The latter is imaginary self-dual

$$\widetilde{\Omega}_{mnp} = \frac{i}{6}\sqrt{\rho_6}\;\varepsilon_{mnpqrs}\widetilde{\Omega}^{qrs} \tag{A.4}$$

and obeys

$$\widetilde{\Omega} \wedge \omega = \widetilde{\Omega} \wedge \omega^* = 0 \,. \tag{A.5}$$

As follows from (A.2), the two globally defined spinors are related via

$$\eta_{2+} = -\frac{1}{2}K^m\gamma_m\eta_{1-} \,. \tag{A.6}$$

We also note the following relations,

$$\begin{aligned}
0 &= (\Pi^+)_m{}^n\gamma_n\eta_{1-} \\
\gamma_{mn}\eta_{1+} &= iJ_{mn}\eta_{1+} + \frac{1}{2}\Omega_{mnp}\gamma^p\eta_{1-} \\
\gamma_{mnp}\eta_{1-} &= -3iJ_{[mn}\gamma_{p]}\eta_{1-} - \Omega^*_{mnp}\eta_{1+} \,.
\end{aligned} \tag{A.7}$$

A useful formula following from (A.6), (A.7) is

$$\gamma^m\eta_{2-} = K^{*m}\eta_{1+} + \frac{i}{2}\omega_{mn}\gamma^n\eta_{1-} \,. \tag{A.8}$$

B. SU(2) structure

Here we give some further useful relations pertaining to the SU(2) structure.

It follows from (A.7) that

$$0 = (\widetilde{\Pi}^+)_m{}^n\gamma_n\eta_{1-} \,, \tag{B.1}$$

where

$$(\widetilde{\Pi}^{\pm})_{mk} := \frac{1}{2}(\widetilde{\rho}_{mk} \mp i\widetilde{J}_{mk}) \tag{B.2}$$

and

$$\widetilde{\rho}_{mk} := \rho_{mk} - \frac{1}{2}(K_m K^*_k + K^*_m K_k)\,. \tag{B.3}$$

Note that

$$K^m \widetilde{\rho}_{mk} = 0\,. \tag{B.4}$$

Some further useful identities are

$$\begin{aligned}
\widetilde{J}_{mn}\widetilde{J}^n{}_k &= -\widetilde{\rho}_{mk} \\
\widetilde{J}_m{}^n \omega_{nk} &= i\omega_{mk} \\
\omega_{mn}\omega^{*nk} &= -4(\widetilde{\Pi}^+)_m{}^k \\
\omega_{mn}\omega^{*ij} &= 8(\widetilde{\Pi}^+)_{[m}{}^i(\widetilde{\Pi}^+)_{n]}{}^j \\
(\widetilde{\Pi}^+)_m{}^k &= (\Pi^+)_m{}^k - \frac{1}{2}K_m K^{*k}\,.
\end{aligned} \tag{B.5}$$

C. SU(2) tensor decompositions

In terms of the SU(2) structure, the form fields of IIA supergravity decompose as follows.

Two-form.

$$B'_{mn} = b_{mn} + b_{[m}K_{n]} + b^*_{[m}K^*_{n]} + ib_1 K_{[m}K^*_{n]}\,, \tag{C.1}$$

where

$$K^i b_{im} = K^i b_i = K^{*i} b_i = 0 \tag{C.2}$$

and

$$\begin{aligned}
K^i B'_{im} &= -b^*_m - ib_1 K_m \\
K^i K^{*j} B'_{ij} &= -2ib_1\,.
\end{aligned} \tag{C.3}$$

Note that b_1 is real. We can further decompose

$$b_{mn} = \widetilde{b}_{mn} + \frac{1}{8}\omega^*_{mn} b_2 + \frac{1}{8}\omega_{mn} b^*_2 + \frac{1}{4}\widetilde{J}_{mn} b_3\,, \tag{C.4}$$

where $\widetilde{b}_{mn}$ is $(1,1)$ and traceless with respect to $\widetilde{J}_{mn}$, i.e. it transforms in the **3** of SU(2). The scalar b_2 is complex whereas b_3 is real. We have

$$\begin{aligned}
b_2 &= \omega^{mn} b_{mn} \\
b_3 &= \widetilde{J}^{mn} b_{mn}\,.
\end{aligned} \tag{C.5}$$

Finally,

$$b_m = -\frac{1}{4}\omega^*_m{}^i \widetilde{b}_{1i} - \frac{1}{4}\omega_m{}^i \widetilde{b}_{2i}\,, \tag{C.6}$$

where $(\Pi^-)_m{}^n \widetilde{b}_{1n} = (\Pi^+)_m{}^n \widetilde{b}_{2n} = 0$. Both $\widetilde{b}_{1i}$, $\widetilde{b}_{2i}$ transform in the **2** of SU(2). We have

$$\begin{aligned}
\widetilde{b}_{1i} &= \omega_m{}^n b_n \\
\widetilde{b}_{2i} &= \omega^*_m{}^n b_n\,.
\end{aligned} \tag{C.7}$$

Three-form.

$$H_{mnp} = h_{mnp} + h_{[mn}K_{p]} + h^*_{[mn}K^*_{p]} + ih_{[m}K_nK^*_{p]}\,, \tag{C.8}$$

where

$$K^ih_{imn} = K^ih_{im} = K^{*i}h_{im} = K^ih_i = 0 \tag{C.9}$$

and

$$\begin{aligned} K^iH_{imn} &= \frac{2}{3}h^*_{mn} + \frac{2i}{3}h_{[m}K_{n]} \\ K^iK^{*j}H_{ijm} &= -\frac{2i}{3}h_m\,. \end{aligned} \tag{C.10}$$

Note that h_m is real whereas h_{mn} is complex. We can further decompose

$$h_{mnp} = -\frac{3}{32}\omega_{[mn}\omega^*_{p]}{}^i\widetilde{h}_{1i} - \frac{3}{32}\omega^*_{[mn}\omega_{p]}{}^i\widetilde{h}^*_{1i}, \tag{C.11}$$

where $(\Pi^-)_m{}^n\widetilde{h}_{1n} = 0$. We have

$$\widetilde{h}_{1m} = \omega_m{}^i\omega^{*jk}h_{ijk}\,. \tag{C.12}$$

Moreover

$$h_{mn} = \widetilde{h}_{mn} + \frac{1}{8}\omega^*_{mn}h_1 + \frac{1}{8}\omega_{mn}h_2 + \frac{1}{4}\widetilde{J}_{mn}h_3\,, \tag{C.13}$$

where $\widetilde{h}_{mn}$ is complex and it is $(1,1)$ and traceless with respect to $\widetilde{J}_{mn}$. The scalars $h_{1,2,3}$ are complex. We have

$$\begin{aligned} h_1 &= \omega^{mn}h_{mn} \\ h_2 &= \omega^{*mn}h_{mn} \\ h_3 &= \widetilde{J}^{mn}h_{mn}\,. \end{aligned} \tag{C.14}$$

Finally,

$$h_m = -\frac{1}{4}\omega^*_m{}^i\widetilde{h}_{2i} - \frac{1}{4}\omega_m{}^i\widetilde{h}^*_{2i}\,, \tag{C.15}$$

where $(\Pi^-)_m{}^n\widetilde{h}_{2n} = 0$. We have

$$\widetilde{h}_{2i} = \omega_m{}^nh_n\,. \tag{C.16}$$

Four-form.

$$G_{mnpq} = g_{mnpq} + g_{[mnp}K_{q]} + g^*_{[mnp}K^*_{q]} + ig_{[mn}K_pK^*_{q]}\,, \tag{C.17}$$

where

$$K^ig_{imnp} = K^ig_{imn} = K^{*i}g_{imn} = K^ig_{im} = 0 \tag{C.18}$$

and

$$\begin{aligned} K^iG_{imnp} &= -\frac{1}{2}g^*_{mnp} - \frac{i}{2}g_{[mn}K_{p]} \\ K^iK^{*j}G_{ijmn} &= -\frac{i}{3}g_{mn}\,. \end{aligned} \tag{C.19}$$

Note that g_{mnpq}, g_{mn} are real whereas g_{mnp} is complex. We can further decompose

$$g_{mnpq} = \frac{3}{8}\widetilde{J}_{[mn}\widetilde{J}_{pq]}g_1\,, \tag{C.20}$$

where the scalar g_1 is real. We have

$$g_1 = \widetilde{J}^{mn}\widetilde{J}^{pq}g_{mnpq}\,. \tag{C.21}$$

Moreover

$$g_{mnp} = -\frac{3}{32}\omega_{[mn}\omega^{*}_{p]}{}^{i}\widetilde{g}_{1i} - \frac{3}{32}\omega^{*}_{[mn}\omega_{p]}{}^{i}\widetilde{g}_{2i}\,, \tag{C.22}$$

where $(\Pi^{-})_m{}^n\widetilde{g}_{1n} = (\Pi^{+})_m{}^n\widetilde{g}_{2n} = 0$. We have

$$\begin{aligned}\widetilde{g}_{1m} &= \omega_m{}^i\omega^{*jk}g_{ijk}\\ \widetilde{g}_{2m} &= \omega^{*}_m{}^i\omega^{jk}g_{ijk}\,.\end{aligned} \tag{C.23}$$

Finally,

$$g_{mn} = \widetilde{g}_{mn} + \frac{1}{8}\omega^{*}_{mn}g_2 + \frac{1}{8}\omega_{mn}g_2^{*} + \frac{1}{4}\widetilde{J}_{mn}g_3\,, \tag{C.24}$$

where $\widetilde{g}_{mn}$ is real and it is traceless with respect to $\widetilde{J}_{mn}$. The scalar g_2 is complex whereas g_3 is real. We have

$$\begin{aligned}g_2 &= \omega^{mn}g_{mn}\\ g_3 &= \widetilde{J}^{mn}g_{mn}\,.\end{aligned} \tag{C.25}$$

D. SU(2) supersymmetry reduction

Using (A.6) and the decompositions of section C, it follows that conditions (6.4), (6.5) can be cast in the form

$$U_m\eta_{1+} + U_{mn}\gamma^n\eta_{1-} = 0\,, \tag{D.1}$$

whereas conditions (6.6)–(6.9) can be written as

$$V\eta_{1+} + V_m\gamma^m\eta_{1-} = 0\,, \tag{D.2}$$

for some U_m, U_{mn}, V, V_m. The explicit expressions for the U's and V's can be readily read off from the following decompositions in terms of irreducible SU(2) representations:

Two-form.

$$\begin{aligned}(\gamma_m{}^{np}B'_{np} - 14\gamma^p B'_{mp})\eta_{1-} = &\left\{-iK^{*}_m b_2^{*} - 2i\widetilde{b}_{2m}\right\}\eta_{1+} +\\ &+\left\{(2b_1 - 3b_3)\widetilde{J}_{mn} - 14\widetilde{b}_{mn} - 2i\widetilde{J}_m{}^i\widetilde{b}_{in} - \frac{3}{2}b_2^{*}\omega_{mn} -\right.\\ &\quad - \frac{3}{2}K_m\omega_n{}^i\widetilde{b}_{2i} - -2K^{*}_m\omega_n{}^i\widetilde{b}^{*}_{1i} +\\ &\quad\left. +K_n\left(2\omega^{*}_m{}^i\widetilde{b}_{1i} + \frac{3}{2}\omega_m{}^i\widetilde{b}_{2i}\right) + iK_nK^{*}_m\left(7b_1 - \frac{1}{2}b_3\right)\right\}\gamma^n\eta_{1-}\,,\end{aligned} \tag{D.3}$$

JHEP08(2005)056

$$(\gamma_m{}^{np} B'_{np} - 14\gamma^p B'_{mp})\eta_{2-} = \left\{iK^*_m(14b_1+b_3) + 3\omega^*_m{}^i\widetilde{b}_{1i} + 4\omega_m{}^i\widetilde{b}_{2i}\right\}\eta_{1+} + \tag{D.4}$$
$$+\left\{-3b_2\widetilde{J}_{mn} + \left(b_1+\frac{3}{2}b_3\right)\omega_{mn} - i\omega_m{}^i\widetilde{b}_{in} - 7i\omega_n{}^i\widetilde{b}_{im} - \right.$$
$$\left. - 3iK_m\widetilde{b}_{1n} - 4iK^*_m\widetilde{b}^*_{2n} - iK_n\widetilde{b}_{1m} - \frac{i}{2}b_2K_nK^*_m\right\}\gamma^n\eta_{1-}$$

and

$$B'_{mn}\gamma^{mn}\eta_{1+} = i(b_3+2b_1)\eta_{1+} + \left\{-i\widetilde{b}^*_{2m} - \frac{i}{2}b_2K_m\right\}\gamma^m\eta_{1-}\,, \tag{D.5}$$
$$B'_{mn}\gamma^{mn}\eta_{2+} = ib^*_2\eta_{1+} + \left\{\frac{1}{2}\omega_m{}^i\widetilde{b}^*_{1i} + iK_m\left(\frac{1}{2}b_3 - b_1\right)\right\}\gamma^m\eta_{1-}\,. \tag{D.6}$$

Three-form.

$$H_{npq}(\gamma_m{}^{npq} - 9\delta_m{}^n\gamma^{pq})\eta_{1+} =$$
$$= \left\{-3\widetilde{h}_{1m} + \frac{3}{2}\widetilde{h}^*_{1m} + 2i\omega_m{}^i\widetilde{h}^*_{2i} + i\omega^*_m{}^i\widetilde{h}_{2i} - 4ih_3K_m - 2ih^*_3K^*_m\right\}\eta_{1+} +$$
$$+\left\{3K_m\widetilde{h}_{2n} + K_n\widetilde{h}_{2m} - \frac{3i}{8}K_m\omega_n{}^i\widetilde{h}^*_{1i} - \frac{9i}{8}K_n\omega_m{}^i\widetilde{h}^*_{1i} + 2ih_1K_mK_n + ih^*_2K_nK^*_m - \right.$$
$$\left. - 2h^*_2\,\widetilde{J}_{mn} + h^*_3\omega_{mn} - 2i\omega_m{}^j\widetilde{h}^*_{jn} - 6i\omega_n{}^j\widetilde{h}^*_{jm}\right\}\gamma^n\eta_{1-}\,, \tag{D.7}$$
$$H_{npq}(\gamma_m{}^{npq} - 9\delta_m{}^n\gamma^{pq})\eta_{2+} =$$
$$= \left\{-2\widetilde{h}^*_{2m} + \frac{9i}{4}\omega^*_m{}^i\widetilde{h}_{1i} - 4iK_mh_2 - 2iK^*_mh^*_1\right\}\eta_{1+} + \tag{D.8}$$
$$+\left\{\frac{3}{2}K_n\widetilde{h}^*_{1m} - \frac{3}{4}K_n\widetilde{h}_{1m} - \frac{3}{4}K_m\widetilde{h}_{1n} + \frac{3i}{2}K_m\omega_n{}^i\widetilde{h}^*_{2i} - \frac{i}{2}K_n\omega_m{}^i\widetilde{h}^*_{2i} - iK_n\omega^*_m{}^i\widetilde{h}_{2i} - \right.$$
$$\left. - 2ih_3K_mK_n - ih^*_3K_nK^*_m + 2h^*_3\,\widetilde{J}_{mn} + 4i\widetilde{J}_m{}^j\widetilde{h}^*_{jn} + \omega_{mn}h^*_1 + 12\widetilde{h}^*_{mn}\right\}\gamma^n\eta_{1-}$$

and

$$H_{mnp}\gamma^{mnp}\eta_{1-} = -2ih_2\eta_{1+} + \left\{\frac{i}{2}\omega_m{}^i\widetilde{h}^*_{2i} - \frac{3}{4}\widetilde{h}_{1m} - ih_3K_m\right\}\gamma^m\eta_{1-}\,, \tag{D.9}$$
$$H_{mnp}\gamma^{mnp}\eta_{2-} = 2ih_3\eta_{1+} + \left\{\frac{3i}{8}\omega_m{}^i\widetilde{h}^*_{1i} - \widetilde{h}_{2m} - ih_1K_m\right\}\gamma^m\eta_{1-}\,. \tag{D.10}$$

Four-form.

$$\left(\gamma_m{}^{npqr}G_{npqr} - \frac{20}{3}\gamma^{pqr}G_{mpqr}\right)\eta_{1-} =$$
$$= \left\{\frac{10}{3}K^*_mg^*_2 - \frac{5i}{2}\omega^*_m{}^i\widetilde{g}_{1i}\right\}\eta_{1+} +$$
$$+\left\{i\left(5g_1 + \frac{2}{3}g_3\right)\widetilde{J}_{mn} + \frac{20i}{3}\widetilde{g}_{mn} - 4\widetilde{J}_m{}^i\widetilde{g}_{in} + \frac{i}{3}g^*_2\omega_{mn} - \right. \tag{D.11}$$
$$\left. - \frac{1}{2}K_m\widetilde{g}_{1n} - 2K^*_m\widetilde{g}^*_{2n} + K_n\left(\frac{1}{2}\widetilde{g}_{1m} - 2\widetilde{g}_{2m}\right) + K_nK^*_m\left(\frac{5}{3}g_3 - \frac{3}{2}g_1\right)\right\}\gamma^n\eta_{1-}\,,$$

$$\left(\gamma_m{}^{npqr}G_{npqr} - \frac{20}{3}\gamma^{pqr}G_{mpqr}\right)\eta_{2-} =$$
$$= \left\{-K_m^*\left(3g_1 + \frac{10}{3}g_3\right) - 4\widetilde{g}_{1m} + \widetilde{g}_{2m}\right\}\eta_{1+} +$$
$$+ \left\{\frac{2i}{3}g_2\widetilde{J}_{mn} + \left(\frac{5i}{2}g_1 - \frac{i}{3}g_3\right)\omega_{mn} - 2\omega_m{}^i\widetilde{g}_{in} - \frac{10}{3}\omega_n{}^i\widetilde{g}_{im} + \right.$$
$$\left. + \frac{i}{4}K_m\omega_n{}^i\widetilde{g}_{2i} + iK_m^*\omega_n{}^i\widetilde{g}_{1i}^* - \frac{5i}{4}K_n\omega_m{}^i\widetilde{g}_{2i} + \frac{5}{3}g_2K_nK_m^*\right\}\gamma^n\eta_{1-} \qquad \text{(D.12)}$$

and

$$G_{mnpq}\gamma^{mnpq}\eta_{1+} = -(3g_1 + 2g_3)\eta_{1+} + \left\{\frac{3i}{4}\omega_m{}^i\widetilde{g}_{1i}^* + g_2K_m\right\}\gamma^m\eta_{1-}\,, \qquad \text{(D.13)}$$
$$G_{mnpq}\gamma^{mnpq}\eta_{2+} = -2g_2^*\eta_{1+} + \left\{\frac{3}{2}\widetilde{g}_{2m}^* + K_m\left(\frac{3}{2}g_1 - g_3\right)\right\}\gamma^m\eta_{1-}\,. \qquad \text{(D.14)}$$

Finally, a derivative (∂_m) on X_6 will be decomposed as

$$\partial_m = \widetilde{\partial}_m + \frac{1}{2}K_mK^{*n}\partial_n + \frac{1}{2}K_m^*K^n\partial_n\,, \qquad \text{(D.15)}$$

so that

$$\iota_K\widetilde{\partial} = 0\,. \qquad \text{(D.16)}$$

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JHEP08(2005)056

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